

Forward jets in the colour-dipole model

S. Munier^{1,†}

¹ *Service de Physique Théorique, CEA/Saclay, F-91191 Gif-sur-Yvette Cedex, France.*

We show that a forward jet with large transverse momentum in an onium-onium collision is a hard probe which can be effectively characterized by a colour-dipole distribution at the time of the interaction. The dipole distribution is computed, and compared to its counterpart for a virtual photon in the initial state. We find that while in the photon case, the tail of large sizes is exponentially cut-off, it contributes sizeably in the forward-jet case, which signs the sensitivity of observables based on such events to the infrared region. Moreover, a direct probabilistic interpretation of the dipole distribution fails since it takes negative values in the large size region.

1 Introduction.

The physics at HERA has proved the successes of (re-summed) perturbative QCD in describing many observables accurately measured there: inclusive ones like the structure function F_2 , but also more exclusive ones, like the diffractive structure function F_2^D , or heavy meson production. The justification for relying on a perturbative development is that the deep-inelastic scattering process naturally provides a well-controlled hard scale given by the photon virtuality Q^2 , which makes the effective strong coupling constant $\alpha_s(Q^2)$ small enough. On the one hand, the physics of these observables is usually well described by the renormalization group evolution [1, 2, 3] between a lower scale Q_0^2 at which the proton parton densities are parametrized and the scale Q^2 . On the other hand, the cross-section for the events selected with the requirement that a forward jet of transverse momentum q^2 of the order of Q^2 be present in the final state is seemingly not described by a straightforward DGLAP evolution (see ref.[4] and references therein): as a matter of fact, these Regge-like kinematics are expected to select the BFKL dynamics [5, 6, 7].

In $p-\bar{p}$ collisions at the Tevatron, no hard scale is provided by the initial state. However, it can be generated in the scattering and manifests itself in the final state in the form of a jet with a large transverse momentum. Events of this class are also accessible to a perturbative QCD interpretation.

In this letter, we focus on high-energy onia (massive $q\bar{q}$ states) collisions, as a model for $p-\bar{p}$ collisions at high energy when high-mass scales are selected by forward jets. The inclusive cross-sections for onia collisions have been described using a dipole cascade modelling the rapidity evolution of the $q\bar{q}$ pairs before their interaction [8]. Here we require that at least the first (most forward) gluon which goes to the final state has its transverse momentum larger than a scale μ ; this gluon becomes a forward-jet,

and we interpret it as an effective colour-dipole distribution present at the time of the scattering.

In section 2, we detail the modelisation that we adopt for the forward jet. We show in section 3 how we can extract to double-leading logarithmic (DLL) approximation, the dipole content of such an object. Section 4 contains our conclusions and outlook.

2 Emission of a forward gluon in the final state.

The onium-onium forward scattering amplitude involves the exchange of two gluons between the initial onia. At high-energy, one has to take into account the possibility of multiple splittings of these t -channel gluons. This can be done either by (k_\perp) factorizing [9] a BFKL-like ladder between the bare onia, or equivalently by computing the two-gluon exchange diagram between the onia dressed by an arbitrary number of soft “sea” gluons. The latter approach inspired the colour-dipole model of ref.[8]. The equivalence between these two methods was shown on different features of both pictures in ref.[10, 11, 12, 13].

In this section, we shall derive in a t -channel picture the gluon density f which is to be considered in an interacting “onium+forward-jet” system as the starting-point of a BFKL evolution. The next section will be devoted to the interpretation of the obtained density as an effective primordial dipole density inside the forward jet.

We have to compute the “dipole+(virtual)gluon $\rightarrow \bar{q}q$ +gluon” amplitude, where the initial-state dipole of radius \vec{r} is part of an onium, and the final-state gluon has a transverse momentum \vec{q} . The modulus $q \equiv |\vec{q}|$ is larger than a given scale μ . Having in mind the fact that in a physical process the initial-state dipole will be part of a hadron instead of an onium, we will consider in the following that $\mu \gg 1/r$ whenever needed for technical purpose.

[†]Electronic address: munier@spht.saclay.cea.fr

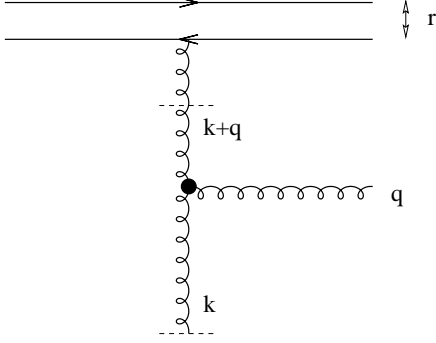


FIG. 1: One of the diagrams contributing to the dipole $g^* \rightarrow g(\text{forward}) + \dots$ cross-section. The horizontal dashed lines represent the points where k_\perp -factorization is applied.

We start with the (virtual)gluon-dipole cross-section $\hat{\sigma}_{g-d}$ which defines the gluon density inside a dipole at lowest order in $\bar{\alpha} \equiv \alpha_s N_c / \pi$. It reads (see ref.[12]):

$$f^0(\vec{k}^2) = \frac{\hat{\sigma}_{g-d}}{\vec{k}^2} = \frac{\bar{\alpha}}{\vec{k}^2} \left(2 - e^{i\vec{k} \cdot \vec{x}} - e^{-i\vec{k} \cdot \vec{x}} \right). \quad (1)$$

It can be expressed as an inverse Mellin-transform in the transverse plane:

$$f^0(\vec{k}^2) = \frac{4\bar{\alpha}}{\vec{k}^2} \int \frac{d\sigma}{2i\pi} (kr)^{2\sigma} v(\sigma), \quad (2)$$

where v is interpreted as the well-known dipole-gluon “vertex” in the Mellin space:

$$v(\sigma) = \frac{2^{-2\sigma-1}}{\sigma} \frac{\Gamma(1-\sigma)}{\Gamma(1+\sigma)}. \quad (3)$$

We then factorize the emission of a real gluon. It can be computed directly in the high-energy limit by evaluating the relevant graphs (one of them is pictured in fig.1), but it is also convenient to see it as one step of the BFKL ladder. The initial gluon density f^0 and the density f after emission of a gluon are related through the formula:

$$f(x, \vec{k}^2) = \bar{\alpha} \left(\log \frac{1}{x} \right) \int \frac{d^2 \vec{q}}{\pi \vec{q}^2} \theta(\vec{q}^2 - \mu^2) f^0(|\vec{k} + \vec{q}|^2), \quad (4)$$

which is the lowest order (in α_s) BFKL equation written in an unfolded form (see for instance ref.[14]). The variable x is proportional to $|t|/s$, where s and t are the ordinary Mandelstam variables for the reaction. In deep-inelastic scattering, x would stand for the Bjorken variable.

Let us write the Mellin-transform of eq.(4):

$$\begin{aligned} \frac{h(\gamma)}{\gamma} &\equiv \int_0^\infty \frac{d^2 \vec{k}}{\pi \vec{k}^2} |\vec{k}|^{2\gamma} f(x, \vec{k}^2) = \bar{\alpha} \left(\log \frac{1}{x} \right) \times \\ &\times \int \frac{d^2 \vec{q}}{\pi \vec{q}^2} \theta(\vec{q}^2 - \mu^2) \int \frac{d^2 \vec{k}}{\pi \vec{k}^2} |\vec{k}|^{2\gamma} f^0(|\vec{k} + \vec{q}|^2), \end{aligned} \quad (5)$$

where we used similar notations as in ref.[9], although we have not performed the Mellin-transform with respect to x . The variable γ describes a path $]\gamma_0 - i\infty, \gamma_0 + i\infty[$ in the complex plane, with $0 < \text{Re } \gamma_0 < 1/2$. Inserting eqs.(1)-(3) into eq.(5), it follows that:

$$\begin{aligned} \frac{h(\gamma)}{\gamma} &= 4\bar{\alpha}^2 \left(\log \frac{1}{x} \right) \int \frac{d\sigma}{2i\pi} v(\sigma) r^{2\sigma} \times \\ &\times \int \frac{d^2 \vec{q}}{\pi \vec{q}^2} \theta(\vec{q}^2 - \mu^2) \int \frac{d^2 \vec{k}}{\pi \vec{k}^2} |\vec{k}|^{2\gamma-2} |\vec{k} - \vec{q}|^{2\sigma-2}. \end{aligned} \quad (6)$$

The integration over \vec{k} can easily be performed switching to complex variables and using the well-known identity (see [15] and references therein):

$$\begin{aligned} \int \frac{dz d\bar{z}}{2i} |z|^{2\alpha-2} |1-z|^{2\beta-2} \\ = \pi \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \frac{\Gamma(1-\alpha-\beta)}{\Gamma(1-\alpha)\Gamma(1-\beta)}, \end{aligned} \quad (7)$$

valid for $\text{Re } \alpha, \text{Re } \beta > 0$ and $\text{Re}(\alpha+\beta) < 2$. The result reads:

$$\begin{aligned} \frac{h(\gamma)}{\gamma} &= 2\bar{\alpha}^2 \left(\log \frac{1}{x} \right) \frac{\Gamma(\gamma)}{\Gamma(1-\gamma)} \mu^{2\gamma-2} \times \\ &\times G_{35}^{40} \left(\begin{matrix} 1, 1, 2-\gamma \\ 0, 0, 1-\gamma, 1-\gamma, 1-\gamma \end{matrix} \middle| \left(\frac{\mu r}{2} \right)^2 \right), \end{aligned} \quad (8)$$

where the Meijer-function G_{35}^{40} (which arguments will be abbreviated in the following) writes:

$$G_{35}^{40}(\gamma, \mu r) = \int_{\mathcal{C}} \frac{d\sigma}{2i\pi} \left(\frac{\mu r}{2} \right)^{2\sigma} \frac{1}{\sigma^2(1-\gamma-\sigma)} \frac{\Gamma(1-\gamma-\sigma)}{\Gamma(\gamma+\sigma)}, \quad (9)$$

the contour of integration \mathcal{C} being defined on fig.2. The function $h(\gamma)/\gamma$ can be seen as the coefficient-function in the sense of ref.[9], for the onium+forward-jet system.

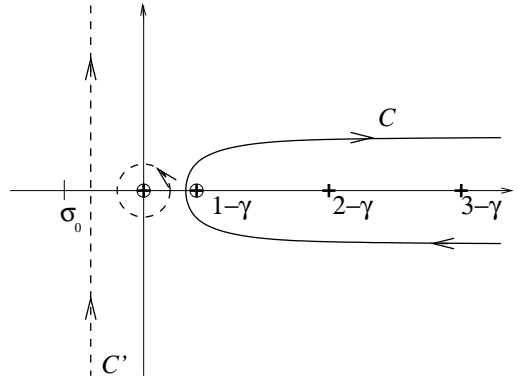


FIG. 2: Contour of integration. Crosses: poles of the integrand. Circles: double poles. Dashed line: contour of integration after deformation.

3 Interpretation in the colour-dipole model.

In ref.[13], a relation was established between the coefficient-function $h(\gamma)/\gamma$ of a virtual photon and its corresponding squared wave-function $\varphi(\gamma)$ on a dipole basis. We shall make use of it in the present context of semi-exclusive factorization to extract the dipole content of the forward jet from the coefficient-function found above, at DLL accuracy. The double-logs should manifest themselves through factors like $\bar{\alpha} \log(1/x) \log(r/r_0)$, where r_0 is a characteristic size for the final state. The relationship between the squared wave-function and the coefficient-function reads:

$$\varphi(\gamma) = \frac{1}{\bar{\alpha}} \frac{h(\gamma)}{\gamma} \frac{1}{v(1-\gamma)}. \quad (10)$$

An intuitive way of understanding this relation could be the following: dividing the coefficient-function pictured in fig.1 by the factor $v(1-\gamma)$ amounts to getting rid of the vertex of the lowest gluon which in this picture is also a gluon-dipole vertex. Hence one ends up with the dipole content of the scattering object:

$$\varphi(\gamma) = 4\bar{\alpha} \left(\log \frac{1}{x} \right) \left(\frac{\mu}{2} \right)^{2\gamma-2} (1-\gamma)^2 \times G_{35}^{40}(\gamma, \mu r). \quad (11)$$

Let us explore the limit $\mu r \gg 1$, in which the forward jet has a transverse momentum much larger than the characteristic scale of the initial onium. The Meijer-function G_{35}^{40} can be approximated in a straightforward manner by picking the pole (at $\sigma=0$) which lies on the left of the integration path. Indeed, the contour \mathcal{C} can be deformed to \mathcal{C}' (see fig.2) since the integral converges on any path such that $\text{Re } \sigma > \sigma_0$, where $\sigma_0 \equiv -1/2 - \text{Re } \gamma$:

$$\begin{aligned} G_{35}^{40}(\gamma, \mu r) &= \int_{\mathcal{C}} \frac{d\sigma}{2i\pi} \left(\frac{\mu r}{2} \right)^{2\sigma} \frac{1}{\sigma^2(1-\gamma-\sigma)} \frac{\Gamma(1-\gamma-\sigma)}{\Gamma(\gamma+\sigma)} \\ &= \frac{\partial}{\partial \sigma} \Big|_{\sigma=0} \left(\left(\frac{\mu r}{2} \right)^{2\sigma} \frac{1}{1-\gamma-\sigma} \frac{\Gamma(1-\gamma-\sigma)}{\Gamma(\gamma+\sigma)} \right) + \\ &+ \int_{\mathcal{C}'} \frac{d\sigma}{2i\pi} \left(\frac{\mu r}{2} \right)^{2\sigma} \frac{1}{\sigma^2(1-\gamma-\sigma)} \frac{\Gamma(1-\gamma-\sigma)}{\Gamma(\gamma+\sigma)}. \end{aligned} \quad (12)$$

The integral taken on the contour \mathcal{C}' is subdominant by some power of $2/(\mu r)$ with respect to the contribution of the double-pole at $\sigma=0$, and one writes:

$$\begin{aligned} G_{35}^{40}(\gamma, \mu r) &= \frac{\Gamma(1-\gamma)}{(1-\gamma)\Gamma(\gamma)} \left\{ 2 \log \frac{\mu r}{2} - \psi(\gamma) - \psi(1-\gamma) + \right. \\ &+ \left. \frac{1}{1-\gamma} \right\} + \{\text{terms suppressed by powers of } 1/(\mu r)\}. \end{aligned} \quad (13)$$

This approximation proves to be very good numerically even for relatively small μr .

Then the squared dipole wave-function reads, in this approximation:

$$\varphi(\gamma) = 4\bar{\alpha} \left(\log \frac{1}{x} \right) \left(\frac{\mu}{2} \right)^{2\gamma-2} \frac{\Gamma(2-\gamma)}{\Gamma(\gamma)} \left(2 \log \frac{\mu r}{2} - \psi(\gamma) - \psi(1-\gamma) + \frac{1}{1-\gamma} \right). \quad (14)$$

We want to obtain an expression for the distribution of dipoles in the system in coordinate space, i.e. as a function of the transverse size ρ . It is given by the inverse-Mellin transform of $\varphi(\gamma)$:

$$\varphi(\rho) = \frac{1}{\rho^2} \int \frac{d\gamma}{2i\pi} \rho^{2\gamma-2} \varphi(\gamma). \quad (15)$$

We obtain the following result:

$$\varphi(\rho) = 8\bar{\alpha} \left(\log \frac{1}{x} \right) \left(\left(\log \frac{r}{\rho} \right) \frac{\mu}{\rho} J_1(\mu\rho) + \frac{1}{\rho^2} J_0(\mu\rho) \right). \quad (16)$$

The second term is not relevant in our approximation, since the limit of large μr selects the DLL; the terms beyond this approximation are not under control. Hence the interaction of the system formed by the initial dipole and the forward gluon can be viewed as a dipole-dipole interaction provided the system is described by the following dipole distribution:

$$\varphi_{\text{DLL}}(\rho) = 8\bar{\alpha} \left(\log \frac{1}{x} \right) \left(\log \frac{r}{\rho} \right) \frac{\mu}{\rho} J_1(\mu\rho). \quad (17)$$

Let us give an interpretation of the various factors in this distribution. First, note that the dependence on the initial dipole size r only appears in the factor $\log(r/\rho)$. This remarkable fact technically results from the combination of the term $\log(\mu r/2)$ and the inverse-Mellin transform of the ψ functions in the inverse-Mellin transform of eq.(14). Physically, the overall factor $2\bar{\alpha} \log(1/x) \log(r/\rho)$ can then be interpreted as the probability of finding a dipole of size ρ inside a dipole of size r . Indeed, in the DLL approximation at lowest order in α_s and assuming an available energy proportional to $1/x$, the probability of finding a dipole of size $|\vec{\varrho}|$ between $|\vec{\rho}|$ and $|\vec{r}|$ inside a dipole of size \vec{r} reads [8]:

$$\begin{aligned} &\bar{\alpha} \left(\log \frac{1}{x} \right) \int_{\rho^2}^{r^2} \frac{d^2 \varrho}{\pi} \frac{\vec{r}^2}{\vec{\varrho}^2 (\vec{r} - \vec{\varrho})^2} \\ &\simeq \bar{\alpha} \left(\log \frac{1}{x} \right) \int_{\rho^2}^{r^2} \frac{d^2 \varrho}{\pi \vec{\varrho}^2} = 2\bar{\alpha} \log \frac{1}{x} \log \frac{r}{\rho}, \end{aligned} \quad (18)$$

where the second approximate equality holds when $\varrho \ll r$. Dividing out this factor in eq.(17) leads to the universal dipole content of the gluon radiated into the final state. We normalize the first moment of the obtained

squared wave-function to unity¹ (i.e. $\phi(\gamma = 1/2) \equiv 1$), and so we are led to:

$$\phi(\rho) = \frac{\mu^2}{\rho} J_1(\mu\rho) . \quad (19)$$

Note that this squared wave-function for which we provide a derivation here is exactly the one (integrated over the energy-share variable z and properly normalized) that was postulated in ref.[16]. To see this, we only need to recall that the Mellin transform of the function $J_1(\mu\rho)/\rho$ reads:

$$\int_0^\infty \frac{d^2\vec{\rho}}{\pi} |\vec{\rho}|^{2-2\gamma} \frac{J_1(\mu\rho)}{\rho} = \mu^{2\gamma-3} 2^{2-2\gamma} \frac{\Gamma(2-\gamma)}{\Gamma(\gamma)} . \quad (20)$$

The behaviour of ϕ is represented in fig.3. It is compared to the behaviour of the transverse virtual photon squared wave-function in $q\bar{q}$ pairs, integrated over the fraction of the photon longitudinal momentum z carried by the quark. This photon wave-function reads:

$$\phi^{\gamma*}(\rho) = \frac{64\mu}{9\pi^3} \int_0^1 dz \mu^2 z(1-z)(z^2 + (1-z)^2) \times \\ \times K_1^2(\mu\rho\sqrt{z(1-z)}) , \quad (21)$$

where the same normalization $\phi^{\gamma*}(\gamma = 1/2) \equiv 1$ has been enforced.

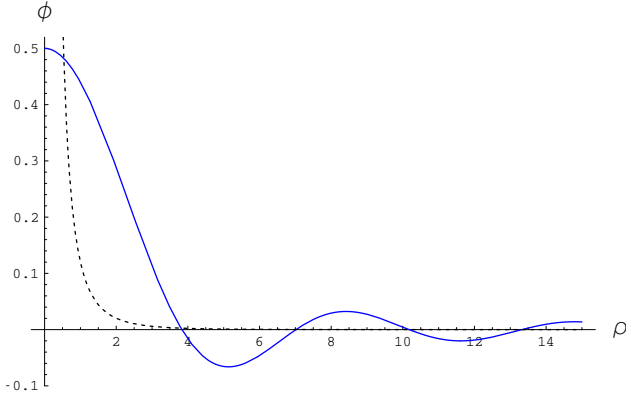


FIG. 3: Dipole density as a function of the transverse size. The scale μ has been set to 1. Continuous line: forward-jet. Dashed line: (transverse) virtual photon.

Some remarks are in order. The squared wave-function ϕ represents the effective distribution of dipoles resulting from a final-state gluon which has its transverse momentum larger than μ . We note that it is slowly decreasing

with ρ ($\sim \rho^{-3/2}$), which means that dipoles of large size occur with a non-negligible probability. The width of the distribution is of order $1/\mu$, but since its behaviour at infinity is only powerlike, μ is not a clean cutoff. It has an oscillatory behaviour, the oscillation length being of the order of $1/\mu$. However, ϕ takes also negative values, which indicates that it does not allow for a direct probabilistic interpretation like in the case of the $q\bar{q}$ -pair distribution $\phi^{\gamma*}$ inside the photon.

4 Conclusions and outlook.

Using a straightforward model and a QCD calculation in the DLL approximation, we have shown that a gluonic hard probe in the final-state can be characterized by a dipole-distribution $\phi(\rho)$. This distribution decreases weakly with ρ and takes negative values. We confirm the (integrated over z) result obtained in ref.[16].

The obtained distribution can now be used to compute processes of several topologies: for instance deep-inelastic scattering on a forward-jet at small- x , $p-\bar{p}$ interactions with two jets in the final-state separated by a large rapidity range. In any case, the dipole formulation for these observables is of the type:

$$\mathcal{O} \propto \int \frac{d\gamma}{2i\pi} \frac{\alpha_s^2}{\gamma^2(1-\gamma)^2} \left(\frac{\mu}{Q_0} \right)^{2\gamma} e^{\bar{\alpha}\chi(\gamma)Y} \times \\ \times \int d^2\rho \rho^{2\gamma} \phi(\rho) \int d^2\rho_t \rho_t^{2-2\gamma} \phi_t(\rho_t) , \quad (22)$$

where $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ is the eigenvalue of the BFKL kernel, and Y the rapidity range between the two probes characterized by the dipole distributions ϕ and ϕ_t . Note that the simplest observable one can compute using this formula is the cross-section σ for the production of dijets of respective transverse momenta larger than k_1 and k_2 in hadronic collisions. It reads:

$$\sigma \propto \frac{\alpha_s^2}{k_1 k_2} \int \frac{d\gamma}{2i\pi} \frac{1}{\gamma(1-\gamma)} \left(\frac{k_1}{k_2} \right)^{2\gamma} e^{\bar{\alpha}\chi(\gamma)Y} , \quad (23)$$

which is the Mueller-Navelet formula [17], modulo some normalization we did not keep precise track of. Hence this little calculation provides a further check of our dipole distribution.

This study may deserve several further investigations and improvements. First of all, we worked only up to terms beyond the DLL approximation. It would be useful to perform a more complete calculation, by computing to leading $\log-1/x$ precision all the graphs of the type of the one in fig.1 which contribute, to see if the dipole factorization still holds.

¹This choice means that ϕ has now the unusual dimension $[\text{mass}]^3$, but it enables a comparison to the photon squared wave-function $\phi^{\gamma*}$, for which the $\gamma = 0$ moment diverges logarithmically.

On the other hand, it would be nice to supplement our indirect method of extracting the dipole content by a more direct calculation in the framework of the colour-dipole model: this would insure the full control of the leading $\log-1/x$, including a correct treatment of the virtual corrections. Although quite straightforward to formulate, the latter calculation exhibits many technical difficulties. Both these proposed improvements deserve more studies.

Acknowledgements:

I thank R. Peschanski and H. Navelet for many useful suggestions and a careful reading of the manuscript.

REFERENCES

- [1] V.N. Gribov and L.N. Lipatov, *Yad. Fiz.* 15 (1972) 781.
- [2] Y.L. Dokshitzer, *Sov. Phys. JETP* 46 (1977) 641.
- [3] G. Altarelli and G. Parisi, *Nucl. Phys.* B126 (1977) 298.
- [4] B. Pötter, (1999), hep-ph/9909320, talk given at the Ringberg Workshop '*New Trends in HERA Physics 1999*'.
- [5] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov. Phys. JETP* 44 (1976) 443.
- [6] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Zh. Eksp. Teor. Fiz.* 72 (1977) 377.
- [7] I.I. Balitsky and L.N. Lipatov, *Sov. J. Nucl. Phys.* 28 (1978) 822.
- [8] A.H. Mueller, *Nucl. Phys.* B415 (1994) 373.
- [9] S. Catani, M. Ciafaloni and F. Hautmann, *Nucl. Phys.* B366 (1991) 135.
- [10] Z. Chen and A.H. Mueller, *Nucl. Phys.* B451 (1995) 579.
- [11] H. Navelet and R. Peschanski, *Nucl. Phys.* B507 (1997) 353, hep-ph/9703238.
- [12] H. Navelet and S. Wallon, *Nucl. Phys.* B522 (1998) 237, hep-ph/9705296.
- [13] S. Munier and R. Peschanski, *Nucl. Phys.* B524 (1998) 377, hep-ph/9802230.
- [14] J. Kwiecinski, A.D. Martin and P.J. Sutton, *Phys. Rev. D* 52 (1995) 1445, hep-ph/9503266.
- [15] J.S. Geronimo and H. Navelet, (2000), math-ph/0003019, to appear.
- [16] R. Peschanski, (1999), hep-ph/9910377, to appear.
- [17] A.H. Mueller and H. Navelet, *Nucl. Phys.* B282 (1987) 727.